

Scientific Measurements

Quantities, Units, and Prefixes

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This is the first in a series of articles on scientific measurements. It lists the fundamental quantities used in scientific measurements and their standard units, introduces derived quantities, denotes some prefixes used to express multiples of units, and shows how to manipulate these unit prefixes. Other topics related to scientific measurements will follow in subsequent articles.

Fundamental Quantities

The **International System of Units** (from French *Système International d'Unités* and abbreviated **SI**) establishes the **fundamental quantities** measured in scientific work. The SI system replaced the metric system about 1960. There are seven fundamental quantities, which are shown below followed in parenthesis by the standard unit and its abbreviation for each quantity.

length (meter, m)	electric current (ampere, A),
mass (kilogram, kg)	amount of substance (mole, mol),
time (second, s)	luminous intensity (candela, cd).
temperature (kelvin, K)	

Students often use the four quantities in the first column. Notice that the abbreviation for the “kelvin” and “ampere,” “K” and “A” respectively, are capital letters. This is because these two units are named after persons, Lord William Thomson Kelvin and André-Marie Ampère. You may have noticed that many thermometers are labeled using the celsius scale and not the kelvin scale. Not all people use only the “accepted standard” but continue to use the older celsius scale because they “are used to it.”

All scientific measurements are *comparisons* of the magnitude of an unknown property, such as length, volume, or force, *to the appropriate standard unit* listed in the table.

Derived Quantities and their Units

Why are these seven called fundamental quantities? Quantities in addition to these seven may be used as comparisons for measurements, but all additional quantities are derived through some combination of these seven fundamental quantities. Quantities obtained by combining fundamental quantities are called **derived quantities**.

Volume is a derived quantity. Every volume can be expressed as the product of three lengths. Suppose that you use a meter stick to measure the volume of a room. If the room is a parallelogram, its volume is measured by measuring its width, depth, and height. The measured volume is found by calculating the product of these three lengths. For example, if the three length measurements are 10.7 m, 8.1 m, and 3.0 m, the volume of the room is reported as 260 m³. The important point to note is that the product of these three lengths has a “numerical answer,” 260, *and* a “unit answer,” m³.

Both the numerical answer and the unit answer are equally important. People sometimes focus their attention only on multiplying numbers to obtain the correct “numerical answer.” They overlook performing the *same operation(s)* on the measured units in order to obtain the correct “unit answer.” The “unit answer” tells the reader the fundamental quantities used to make the measured comparison. In this case, $m \cdot m \cdot m$ equals m^3 . The unit answer for the product is “meters cubed” or “cubic meters” whichever you prefer. Stating the derived unit as a length cubed clearly tells the reader that a measurement of volume is being reported.

This is likely to be confusing for some people because familiar units for volume, such as quart, gallon, and fluid ounce, have nothing in their names that suggest that they are “standing for a length cubed.” This is true; these names were assigned to these volumes long before the SI system was adopted. Nevertheless, there exists, for example, a length that when cubed equals the volume of a quart, but this length is not related to any standard unit used in scientific work.

Density is another derived property. It is measured by dividing a sample’s measured mass by its measured volume. Hence, its “unit answer” is kg per m^3 , which is the fundamental unit of mass divided by the derived unit of volume.

Six familiar derived quantities and their derived units expressed as a combination of fundamental quantities are listed in the following table.

Derived Quantity	Derived SI Unit	Name	Symbol
Volume	m^3	none	none
Density	$kg\ m^{-3}$	none	none
Force	$kg\ m\ s^{-2}$	newton	N
Energy	$kg\ m^2\ s^{-2}$	joule	J (or N m)
Pressure	$kg\ m^{-1}\ s^{-2}$	pascal	Pa (or $N\ m^{-2}$)
Power	$kg\ m^2\ s^{-3}$	watt	W (or $J\ s^{-1}$)

You can observe in this table that the derived units for volume and density have no assigned names. Hence, one may only say, for example, that the volume of gas in the sample is “zero point zero one six cubic meter” and write $0.016\ m^3$. But when reporting a force one might say that the same force is either “six point seven eight kilogram meter per second squared” or “six point seven eight newtons” and write $6.78\ kg\ m\ s^{-2}$ or $6.78\ N$.

Note that the unit for force is written $kg\ m\ s^{-2}$. A single line can be used for this derived unit because the unit second, s, has a “negative 2” exponent. If this term had appeared in the denominator, both a “positive two” exponent and two lines would be needed to write this unit. Often negative exponents are employed when writing units on printed pages in order to avoid having to use two lines to print the unit. The derived units displayed in the table above are shown below written on both one and two lines.

$$Density = \frac{kg}{m^3} = kg\ m^{-3} \quad Force = \frac{kg\ m}{s^2} = kg\ m\ s^{-2} \quad Energy = \frac{kg\ m^2}{s^2} = kg\ m^2\ s^{-2}$$

$$Pressure = \frac{kg}{m\ s^2} = kg\ m^{-1}\ s^{-2} \quad Power = \frac{kg\ m^2}{s^3} = kg\ m^2\ s^{-3}$$

If both 6.78 kg m s^{-2} and 6.78 N are correct, does it make any difference which unit is used? It does. Use “newton” *only if you and your audience* understand that “newton” means kg m s^{-2} .

You can see that names have been assigned to the derived units for force (newton), energy (joule), pressure (pascal), and power (watt). This was likely done for two reasons. First, people find it easier to say “newton” than to say “kilogram meter per second squared.” Also, naming this unit a “newton” honors Isaac Newton as other named units honor James Prescott Joule, Blaise Pascal, and James Watt. You can also see that the name of the unit is not capitalized, but its symbol (N, J, Pa, or W) is capitalized. But, there is a downside to this. Simply memorizing a new word and how to spell it does not indicate that you understand its meaning. Shun memory work; seek understanding.

Unit Prefixes

Sometimes the property measured is much smaller or larger than the standard unit, which makes reporting the measurement cumbersome. For example, the measured thickness of a sheet of paper is 0.00007 m and the distance from Chicago to Denver is $1,600,000 \text{ m}$. The standard unit for length, meter, is used for both these measurements, but it is too large a unit for one and too small a unit for the other. This situation occurs often when making and reporting scientific measurements. The situation is made much better by employing scientific notation to express the measured quantity. Employing scientific notation for the two measurements above:

0.00007 m becomes $7 \times 10^{-5} \text{ m}$, and

$1,600,000 \text{ m}$ becomes $1.6 \times 10^{+6} \text{ m}$.

Doing this makes it easier to record and read these two measurements without error, but it is still cumbersome to write and read the “power of ten” term in the numerical part of each of these measurements. How can this “power of ten” term be eliminated? A step in the right direction would be to assign names to several “power of ten” terms and place that name as a prefix before the unit. A table of often-used “power of ten” prefixes appears below.

Power of Ten Term	Prefix Name	Prefix Symbol
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{+3}	kilo	k
10^{+6}	mega	M
10^{+9}	giga	G
10^{+12}	tera	T

By use of two prefixes in this table, these two measurements may now be written as:

$$0.00007 \text{ m} = 7 \times 10^{-5} \text{ m} = 7 \times 10^{-2} \times 10^{-3} \text{ m} = 7 \times 10^{-2} \text{ mm} \text{ or } \underline{0.07 \text{ mm}}, \text{ and}$$

$$1,600,000 \text{ m} = 1.6 \times 10^{+3} \times 10^{+3} \text{ m} = \underline{1.6 \times 10^{+3} \text{ km}} = \underline{1.6 \text{ Mm}}.$$

These measurements are read “zero point zero seven millimeters” and “one point six times ten to the three kilometers” or “one point six megameters.”

One can see in the table of prefixes that Roman letters are employed for all symbols except micro (10^{-6}), which employs the Greek letter “mu” (μ). Also, all exponential notations equal to mega (10^{+6}) and greater are abbreviated by use of capital letters and those less than mega by lower case letters. Deka, 10^{+1} , is the only prefix that is abbreviated by use of two letters, da, but it does not appear in the above table.

You may have noticed that most of the “powers of ten” prefixes are multiples of a thousand, 10^3 . Hence, not every measurement can be expressed as a number between one and ten accompanied by the use of a “power of ten” prefix. You can see that this is true by considering the two measurements of length mentioned previously. One, 1.6 Mm (megameters), is expressed as a number between one and ten plus a unit having a “power of ten” prefix. But the thickness of the sheet of paper cannot be reported as a number between one and ten plus a unit having a “power of ten” prefix. It must be written as either 0.07 mm or 70 μm . There is no “power of ten” name for 10^{-5} , which would allow it to be expressed as a number between one and ten plus a unit having a “power of ten” prefix standing for 10^{-5} .

Manipulating Unit Prefixes

The prefixes before units stand for “power of ten” terms. A kilometer, km, is 10^{+3} meter or one thousand meters. These prefixes placed before the standard unit of meter change it to a “multiple of that unit,” a thousand times larger (km), a million times larger (Mm), or a billion times larger (Gm). You can even make them smaller. A tenth of a meter (dm), a hundredth of a meter (cm), or a thousandth of a meter (mm), etc. How do we manipulate these unit prefixes to our advantage?

For example, let’s consider a density measurement. Ada reported that the density of iron is $7.8 \times 10^{+3} \text{ kg m}^{-3}$. She used the standard units for the fundamental quantities, but the answer given in her textbook is 7.8 g cm^{-3} . Does Ada’s measurement agree with the value in the book?

Before we go further, let’s get a feeling for the magnitude of a mass of $7.8 \times 10^{+3} \text{ kg}$. The mass of a newborn baby is about 3 kg. A teenager’s mass is about 65 kg. And the fellow that centered the ball at the last Superbowl has a mass of about 130 kg. So, $7.8 \times 10^{+3} \text{ kg}$ is not a mass that a person can pick up. It is greater than that of an automobile!

How big is a cubic meter? It is a cube one meter on each edge, which is about the size of a washing machine. That is a big volume. So, it’s hard to say if Ada’s measurement agrees with the density of iron reported in her textbook without manipulating the prefixes. Let’s do it.

To manipulate prefixes, use what you have learned from the tables above and *always multiply by one* so that the magnitude of the measurement remains *unchanged*. One is any value “over itself.” So, $5/5 = 1$, and $w/w = 1$, and $\pi/\pi = 1$. Next, remember that if anything is multiplied by one, its magnitude does not change. For example, $xyz \cdot (5/5) \cdot (w/w) \cdot (\pi/\pi) \cdot 1 = xyz$. It is that simple! Anything, such as “xyz,” multiplied by “1” numerous times results in a product equal to its initial value, “xyz”!

By use of this information, we can show if Ada and the textbook agree. We shall manipulate Ada’s density unit, kg m^{-3} , in order to change it to g cm^{-3} . This is accomplished in the manipulation expression shown below.

$$7.8 \times 10^{+3} \frac{\text{kg}}{\text{m}^3} \times \left(\frac{10^{+3}}{\text{k}} \right) \times \left(\frac{10^{-2}}{\text{c}} \right)^3 = 7.8 \times 10^{+3} \frac{\text{g}}{\text{m}^3} \times \left(\frac{10^{+3}}{1} \right) \times \left(\frac{10^{-6}}{\text{c}^3} \right) = 7.8 \frac{\text{g}}{\text{cm}^3}$$

Good, Ada's reported measurement, on the left, agrees with the one shown on the right, which is the unit for density used in the book.

Several points should be emphasized about the manipulation of units, which has been shown above and applies to all unit manipulations.

1. Begin every unit manipulation by noting the initial unit (kg m^{-3} above) and the final unit that you want to manipulate the initial unit to become (g cm^{-3} above). Comparing these two units gives the clue to what needs to be done to accomplish a successful manipulation. We see by comparing these two units that we shall have to *eliminate* the "kilo prefix" before the gram in the unit employed by Ada and *insert* a "centi cubed" prefix before the meter cubed in its denominator. We have now identified the two manipulation jobs that we want to accomplish.
2. Second, we shall need *two* well-chosen "ones" to accomplish these two jobs. One to eliminate the "kilo prefix" and the second to insert the "centi cubed" prefix. We chose the first well-chosen "one" so that a "k" appears in its denominator in order to eliminate (or cancel) the "k" in the numerator of the unit employed by Ada. (Remember, k/k is "one," so both these "ks" are eliminated.) The well-chosen "one" that accomplishes this manipulation is enclosed in the first parenthesis on the left in the above manipulation expression.

The second well-chosen "one" should have a "c³" in the denominator so that it can be combined with Ada's volume unit of m^3 . This manipulates the volume unit from m^3 to cm^3 . The well-chosen "one" to accomplish this manipulation, $(10^{-2}/\text{c})$, is enclosed in the second parenthesis in the manipulation expression. But we need a c^3 in the denominator and not just a "c," so we cube this well-chosen "one" in order to manipulate it to obtain what is needed to accomplish the job. Sure, we can cube it, square it, or raise it to any power without changing its value because it equals one. See, well-chosen "ones" are valuable.

3. The last point is a warning. Units that have prefixes are sometimes raised to powers. Two examples are the derived units for area, cm^2 , and volume, cm^3 . Be sure to understand and remember that *the prefix is also raised to that power*. The volume unit is not written as c^3m^3 , but that is what cm^3 means. Put another way, cm^3 means that a centimeter (cm or 0.01 m) is cubed to obtain cm^3 or 10^{-6}m^3 . This is not the same as centi (one hundredth) of a cubic meter, 10^{-2}m^3 .

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