Converting Units

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The SI system has only seven fundamental units, but there are many other units that are used for measurements. For example, to measure length, you might use the fundamental unit, the meter, or you might use centimeters, kilometers, or millimeters to name just a few. How can we compare measurements that are made in different units?

Since any number multiplied by 1 is equal to itself, we can convert between different units using a method that always multiplies by "well-chosen ones." Any value "over itself" is equal to one, so 4/4 = 1, x/x = 1, and π/π = 1. Since 1 meter is the same as 100 centimeters, even 100cm/1m = 1!

To illustrate this method of converting units, consider a density measurement. After doing a density lab, Carly used fundamental units to report the density of iron as 7.8×10^3 kilogram per cubic meter. She checks her textbook to see how her value compares with the accepted value, but the density of iron given in her textbook is 7.8 *grams* per *cubic centimeter*. Does Carly's density of iron agree with the value in the book?

To find out, we shall manipulate Carly's density unit, kg/m^3, to convert it to g/cm^3. This can be accomplished as follows:

(1) At the left end of a horizontal line, write the measurement that you know.

$$\frac{7.8 \times 10^3\ \text{kg}}{\text{m}^3}$$

(2) At the right end of the line, write the desired units.

$$\frac{7.8 \times 10^{+3}\ \text{kg}}{\text{m}^3} \qquad \frac{\text{g}}{\text{cm}^3}$$

(3) In the middle portion of the line, we write our "well-chosen ones" – conversion facts that will help us change from the given unit to the desired unit. These facts are always written as a fraction whose value is 1. In this case, we know that 1000 g equals 1 kg and 1 m equals 100 cm. Consequently, 1000g/1kg = 1 and 1m/100cm = 1, so we can write

$$\frac{7.8 \times 10^{+3}\ \text{kg}}{\text{m}^3} \left| \frac{1000\ \text{g}}{1\ \text{kg}} \right| \frac{1\ \text{m}}{100\ \text{cm}} \left| \frac{1\ \text{m}}{100\ \text{cm}} \right| \frac{1\ \text{m}}{100\ \text{cm}} \Big| \frac{\text{g}}{\text{cm}^3}$$

Notice that each conversion fact is written so that it cancels an unwanted unit, just as you would cancel factors in fractions. This leaves the desired unit. For example, since we do not want kilograms in the final answer, the first conversion fact is written with "1 kg" on the bottom so that it cancels the kg unit on top in the original measurement.

(4) Cancel units wherever possible.

$$\frac{7.8 \times 10^3\ \cancel{\text{kg}}}{\cancel{\text{m}^3}} \left| \frac{1000\ \text{g}}{1\ \cancel{\text{kg}}} \right| \frac{1\ \cancel{\text{m}}}{100\ \text{cm}} \left| \frac{1\ \cancel{\text{m}}}{100\ \text{cm}} \right| \frac{1\ \cancel{\text{m}}}{100\ \text{cm}} \Big| \frac{\text{g}}{\text{cm}^3}$$

(5) Multiply both the numbers and the units from left to right to convert the original measurement to the desired units.

$$\frac{7.8 \times 10^3\ \cancel{\text{kg}}}{\cancel{\text{m}^3}} \left| \frac{1000\ \text{g}}{1\ \cancel{\text{kg}}} \right| \frac{1\ \cancel{\text{m}}}{100\ \text{cm}} \left| \frac{1\ \cancel{\text{m}}}{100\ \text{cm}} \right| \frac{1\ \cancel{\text{m}}}{100\ \text{cm}} \left| \frac{\mathbf{7.8 \times 10^6}\ \text{g}}{\mathbf{10^6}\ \text{cm}^3} \right.$$

(6) Lastly, divide to find the final value. In this case, 7.8×10^6 grams divided by 10^6 cubic centimeters yields 7.8 g/cm^3. Carly's measurement for the density of iron agrees with the one reported in the book.

This method, sometimes referred to as the "Factor-Label" method, will work to convert from any unit to any unit as long as you know the conversion facts!