# *The Enormous Size of Avogadro's Number*

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How big is Avogadro's number? It is a *very* large number! Although it is known with little uncertainty – one reference states that it is 6.0221367±36 x $10^{23}$ – it is not known with absolute accuracy. An uncertainty of ±36 out of 60,221,367 is 6.0 x $10^{-5}$ %. Because Avogadro's number is measured indirectly and cannot be counted, there will always be an uncertainty associated with its determination, but this uncertainty is negligible for our calculations.

The magnitude of Avogadro's number is beyond comprehension, and the following examples are employed to show why this is so.

<u>Example 1</u>

Could a person count at the rate of one integer per second from 1 to Avogadro's number in a lifetime?

If a person lives 80 years, he or she will have lived 2.5 billion seconds (2.5 x $10^9$ s). This number of seconds is on the order of 100,000,000,000,000 or $10^{14}$ times smaller than Avogadro's number!

Now suppose that everyone presently living on earth, 6.6 x $10^9$ people, lived for 80 years. Their combined lifetimes would be only 1.7 x $10^{19}$ s, which is 35,000 times less than Avogadro's number of seconds.

Do you think that the number of seconds in the combined lifetimes of all the people who *ever* lived on earth would require a number as large as Avogadro's number?

<u>Example 2</u>

How about the number of sheets of paper in all the books, newspapers, and magazines ever printed? Is that as large as Avogadro's number?

An IPS textbook contains about 147 sheets of paper (294 pages) per one centimeter of thickness. The thickness of an Avogadro's number of sheets of this paper would be

$$\frac{6.02\ x\ 10^{23} sheets}{147\ sheets\ /\ cm} = 4.10\ x\ 10^{21}\ cm = 4.10\ x\ 10^{16}\ km\ .$$

To see how large a distance this is, consider the speed of light (3 x $10^8$ meters per second or 3 x $10^5$ km/sec). The distance light that travels in one year (a light-year) is given by

$$3\ x\ 10^5 \frac{km}{s}\ x\ 365 \frac{days}{yr}\ x\ 24 \frac{hr}{day}\ x\ 3600 \frac{s}{hr} = 9\ x\ 10^{12} \frac{km}{yr}\ .$$

The time required for light to travel past an Avogadro's number of pages at this incredible speed is

$$\frac{4.10\,x\,10^{16}\ km}{9\,x\,10^{12}\ \frac{km}{yr}} \ = \ 5\,x\,10^{3}\ yr \ = \ 5{,}000\ years\,.$$

Would you suppose that the total number of letters and numbers ever printed, typed, and written on earth would be as large as Avogadro's number?

Example 3

Is the total number of holes in all the screen wire ever made on earth less than Avogadro's number?

The meter was once defined as one ten-millionth of the distance between Earth's equator and its poles. Hence the circumference of Earth is approximately 40 million meters and its radius is about 6.4 x $10^6$ meters or 6.4 x $10^9$ mm. The area of earth's surface, $4\pi r^2$, is calculated to be

$$4\,x\,3.14\,x\left(6.4\,x\,10^{9}\ mm\right)^{2} \ = \ 5.1\,x\,10^{20}\ mm^{2}\ .$$

Suppose that the screen material has square holes that measure about 1 mm on a side. This would mean that it would take a piece of screen wire with 5.1 x $10^{20}$ holes to cover the earth – still far fewer than an Avogadro's number!

Example 4

One of the "Themes for Short Essays" on page 190 of the Eighth Edition of IPS invites students to write a research report entitled "The Number of Sand Grains in a Cup of Sand." Why not extend the same idea to find the edge length of a cube containing an Avogadro's number of sand grains?

Sand grains have different sizes and shapes. Larger grains in a sample probably have diameters between 1 and 2 mm, the smaller grains between 0.2 and 1 mm, and many more tiny grains too small to handle have diameters less than 0.1 mm. To solve this problem, assume that a representative grain of sand is a cube having edges 0.5 mm or 5 x $10^{-4}$ m long. The volume of a single grain of sand is then

$$(5 \text{ x } 10^{-4}\text{ m})^{3} = 1 \text{ x } 10^{-10}\text{ m}^{3} \qquad \text{(to one significant figure).}$$

The volume of a container needed to hold an Avogadro's number of grains of this size is

$$(6 \text{ x } 10^{23}\text{ grains}) \text{ x } (1 \text{ x } 10^{-10}\text{ m}^{3}\text{/grain}) = 6 \text{ x } 10^{13}\text{ m}^{3}.$$

Since the volume of a cube is found by cubing the length of an edge, the edge length is the cube root of the volume.

$$\sqrt[3]{6\,x\,10^{13}\ m^{3}} \ = \ 4\,x\,10^{4}\ m \ = \ 40\,km$$

The container must measure approximately 40 km long by 40 km wide by 40 km high!

Of course, as students discover in IPS 8th edition Experiment 1.4, Measuring Volume by Displacement of Water, the volume occupied by a sample of dry sand is not entirely filled with sand. A volume of dry sand contains about 60% solid sand while air occupies about 40% of the space within the sample. This information may be used to find the volume of a cube filled with dry sand that has Avogadro's number of grains surrounded by air space.

$$\frac{1 m^3\ dry\ sand}{0.6 m^3\ solid\ sand}\ x\ 6 x 10^{13}\ m^3\ solid\ sand\ =\ 1 x 10^{14}\ m^3\ dry\ sand$$

The length of an edge of the cubic container needed to hold this volume is then

$$\sqrt[3]{1 x 10^{14}\ m^3}\ =\ 5 x 10^4\ m\ =\ 50\ km\ =\ 30\ miles.$$

Do not be misled into thinking that this is the "correct" answer. We have made some assumptions along the way. Sand grains are not cubes and they are not all the same size, so this answer is an estimate. Had we assumed that sand grains were twice (x2) as long on an edge, the volume of a cube holding an Avogadro's number of them would have been eight times larger ($x2^3=8$) and the edges of the container would have to be twice as long (100 km). We say that answers like this are good to "an order of magnitude," which is "10 to the first power," $10^1$. So, different people may have answers that differ by 100%, 500%, or even 800%, but they are within an order of magnitude of each other, which is good enough to answer correctly how big the cube would be that holds Avogadro's number of sand grains.